Semi-Lagrange Method for Level Set Based Structural Optimization

XIA Qi

ACAE Department

The Chinese University of Hong Kong, HK

Contents

- *Level set model*
- *Semi-Lagrange method*
    - *Upstream point approximation*
    - *ENO interpolation*
- *Numerical examples*
- *Conclusions*

Level Set Model

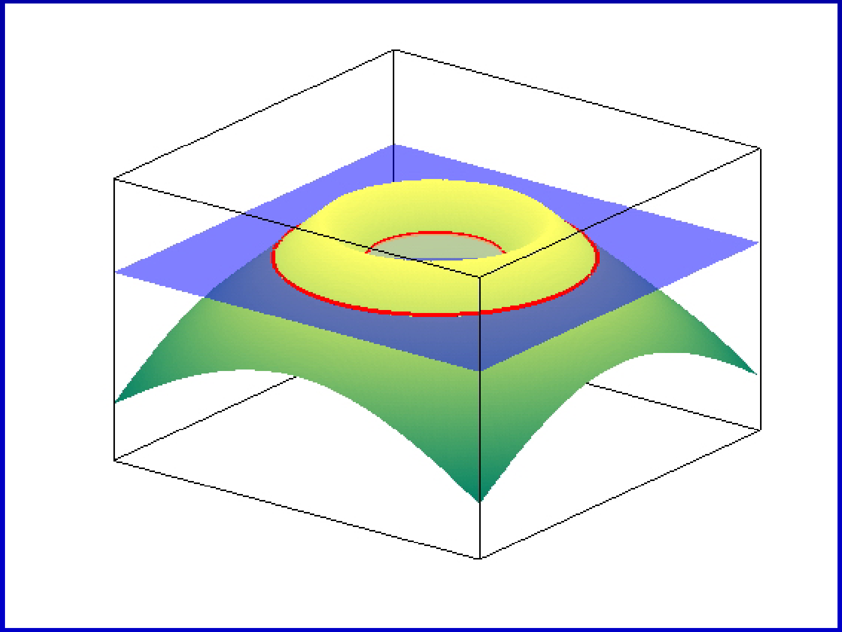
- ***Level Set Methods are numerical algorithms which can simulate the evolution of dynamic implicit interfaces.***

- ***Benefit of level set model for structural optimization***
  - ***Simple treatment of complex topological changes***
  - ***Region-based representation***
  - ***Problem formulation is simple***

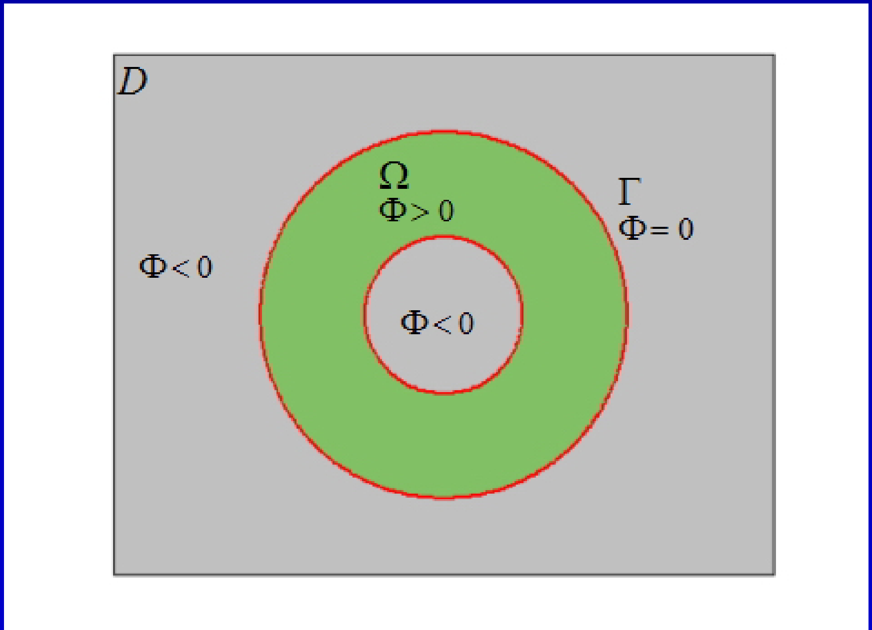
Level Set Model

Fig.1 Level set model and design domain

(a) surface ϕ and the x-y plane



(b) partition of design domain



Level Set Model

- *The process of structural optimization can be described by letting the surface change dynamically in time*

$$\Gamma(t) = \{x(t) : \Phi(x(t), t) = 0\}$$

- *By differentiating both side of the above equation , we get the "level set equation" that models the optimization process*

$$\phi_t + \vec{V} \cdot \nabla \phi = 0$$

Level Set Model

- *Unfortunately, the generally used explicit numerical schemes for level set equation has a severely restricted time step due to Courant-Friedrichs-Lewy (CFL) condition, which makes the solution time consuming.*
- *In optimization, we do not need this kind of smooth propagating front, as long as the final solution obtained is at the global or near-global minimum of the objective function.*
- *Therefore, semi-Lagrange scheme is used to improve the efficiency of the optimization process, which allow us to obtain maximum decent in each iteration and hence to find the optimum structure with as least as possible time steps.*

Semi-Lagrange Method

- *It is a long history in weather forecast science that semi-Lagrange methods are used to speed up the intensive computations.*
- *Recently, it is also introduced into level set equations by Strain to yield a fast and modular algorithm.*
- *The central idea in semi-Lagrange scheme is the "method of characteristics" , in which a upstream point is estimated and the value at the point is interpolated from its neighbors.*

Semi-Lagrange Method

- *Integrating the level set equation, we get*

$$\phi(x,t_2) = \phi(\hat{x},t_1) - \int_P \nabla\phi \cdot (dt + V dx)$$

Where P is an arbitrary path connecting the two space-time points: (x,t_2), $(\hat{x},t_1)$. Different choices of the integration path give rise to three different solution, i.e., Euler solution, Lagrange solution, semi-Lagrange solution.

Semi-Lagrange Method

Fig.2 Euler solution

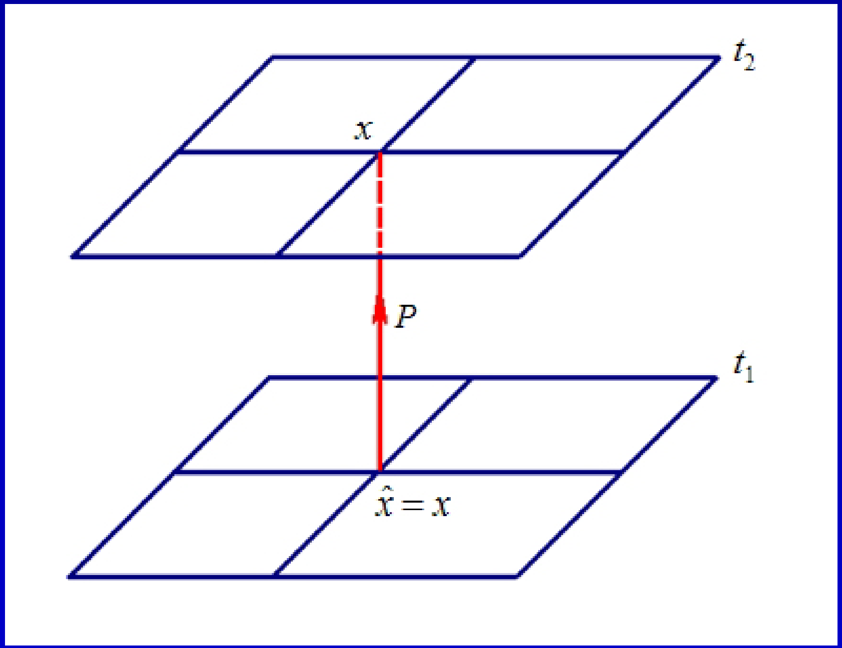
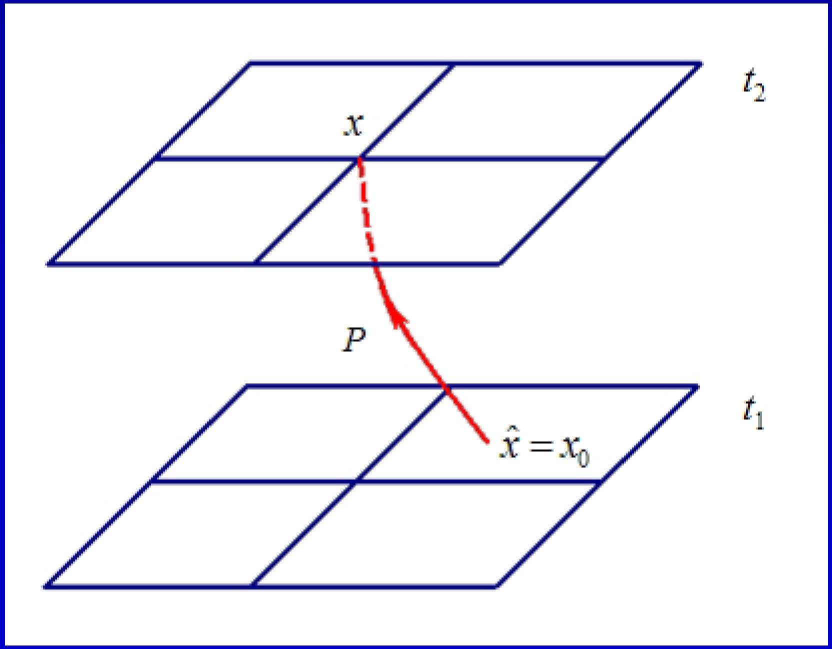
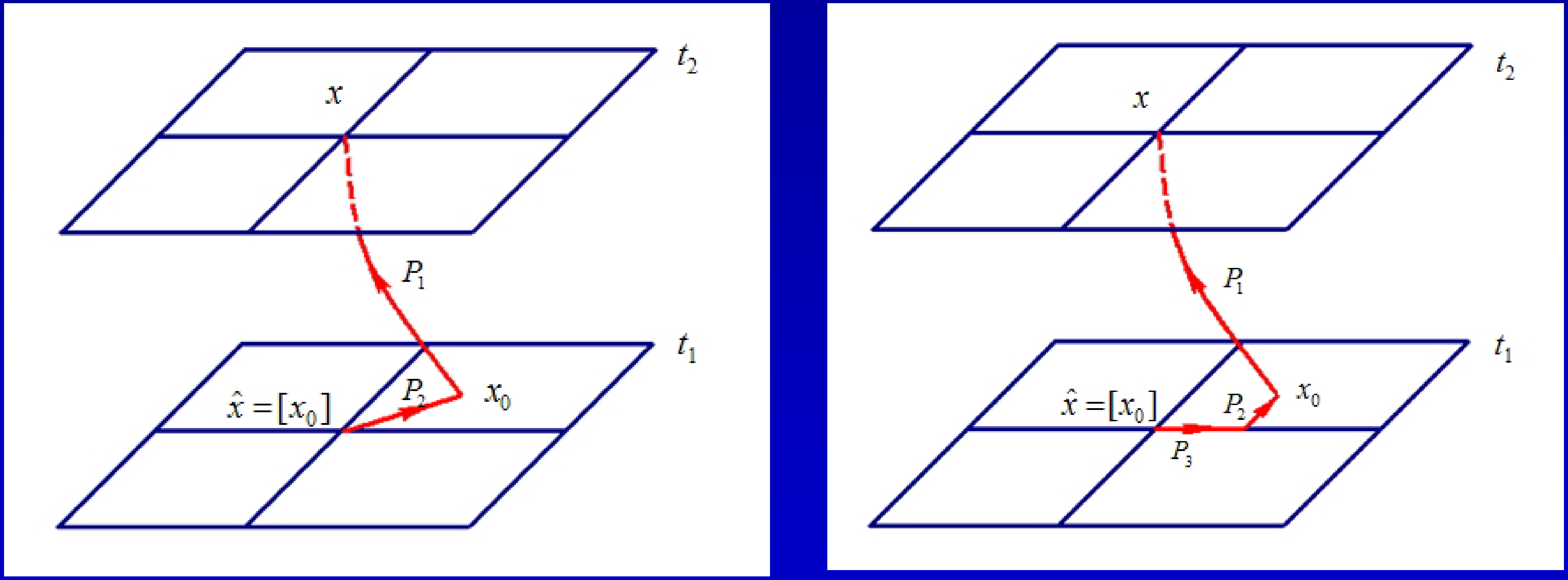


Fig.3 Lagrange solution



Semi-Lagrange Method

Fig.4 Two different semi-Lagrange solution



Upstream point approximation

- ***The simplest upstream point estimation scheme is the so-called "CIR" scheme in the sense that it uses straight lines to approximate the characteristics which would be curves in general.***

- ***The characteristic P is defined as the solution of the following ODE:***

$$\frac{dx}{dt} = V(x,t), \quad P(0) = x_0$$

- ***The CIR scheme approximates the upstream point x_0 by***

$$x_0 = x - \Delta t V(x, t_1)$$

Upstream point approximation

- *In fact, there two approximations here:*
  - *The characteristic connecting point x and x_0 is approximated by a line segment of length* $|\Delta t V(x,t_1)|$
  - *The velocity at point x_0 is approximated by the velocity at point x, i.e.,* $V(x,t_1)$
- *The time step here is usually taken to be three to six times bigger than those allowed by CFL condition.*
- *Actually, to guarantee the numerical stability CFL condition restricts the time step much smaller than that required for accuracy.*

ENO Interpolation

- *Traditionally, a wide stencil is generally used to generate a high order polynomial and therefore to obtain high order of accuracy.*

- *But this strategy will cause some problem in case that the function being interpolated is not smooth within the stencil which is ubiquitous in level set model.*

- *Furthermore, an interpolation should not increase the norm of ϕ too much, otherwise numerical solution would not be stable.*

- *Considering these constraints, Strain propose to use Essentially Non-Oscillatory (ENO) interpolation.*

ENO Interpolation

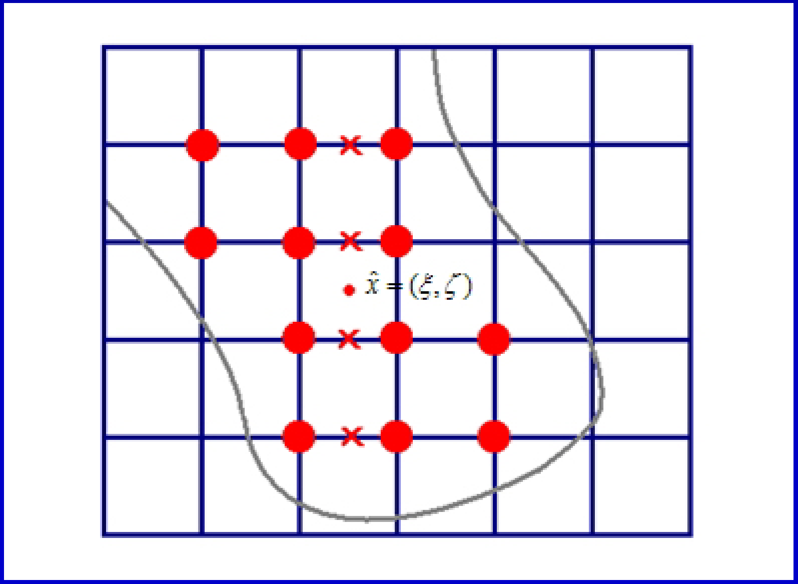
- *Suppose we interpolation a 1D function f(x) to the point x.*
  - *we begin with a stencil that only comprises the two nearest points to x_0, i.e., x_1 on the left side and x_2 on the right side.*
  - *Then we use Newton divided difference to determine the smoothness of the function inside the two possible stencils corresponding to two choices of additional points.*
  - *To get a non-oscillatory result the stencil covering a smoother piece of the function, i.e., with a smaller divided difference, is more preferable.*
  - *Finally, we can use either Lagrange form or Newton form to obtain a second order polynomial and complete the interpolation.*

ENO Interpolation

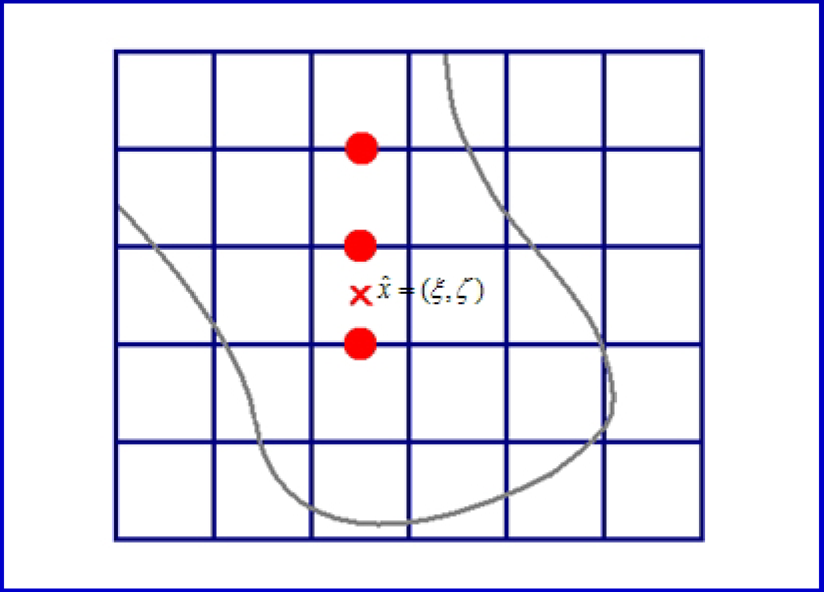
- *In 2D cases, ENO interpolation is carried out by two separate 1D interpolation along each axis.*

Fig.5 ENO interpolation in 2D

(a) Interpolation along horizontal axis



(b) Interpolation along vertical axis



Numerical Examples

- ***Cantilever beam***

Fig.6 The design problem

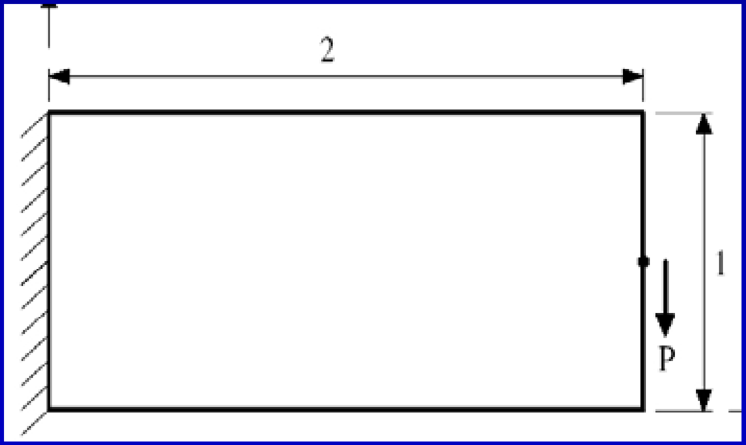
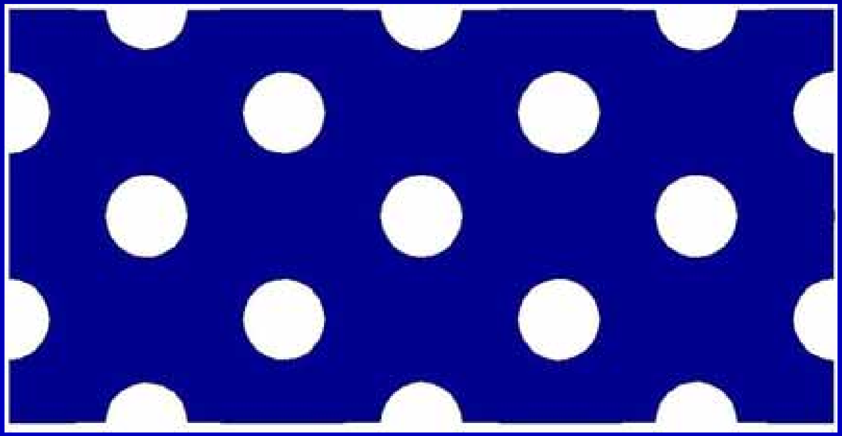
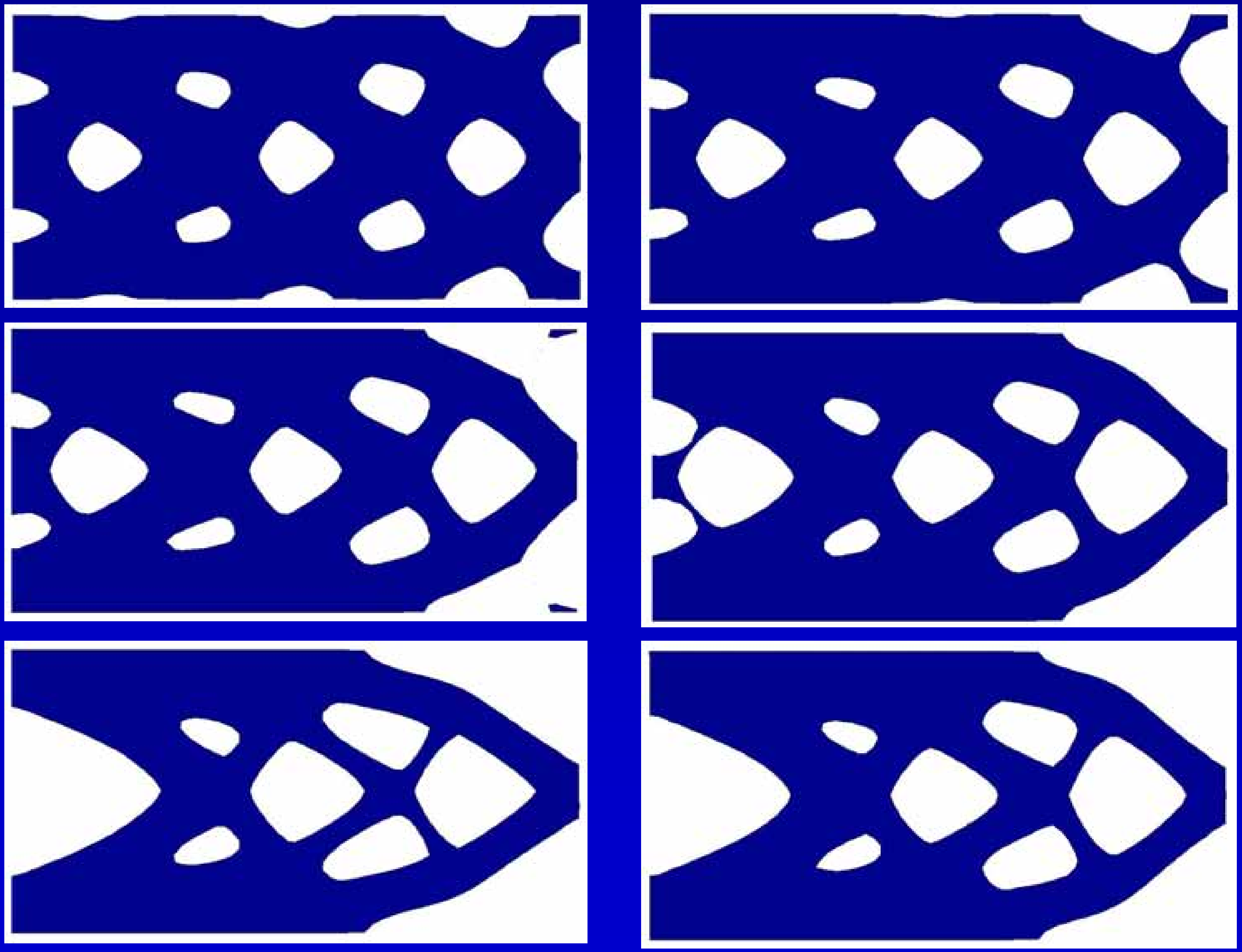


Fig.7 The initial design



Numerical Examples

Fig.8 The optimization process

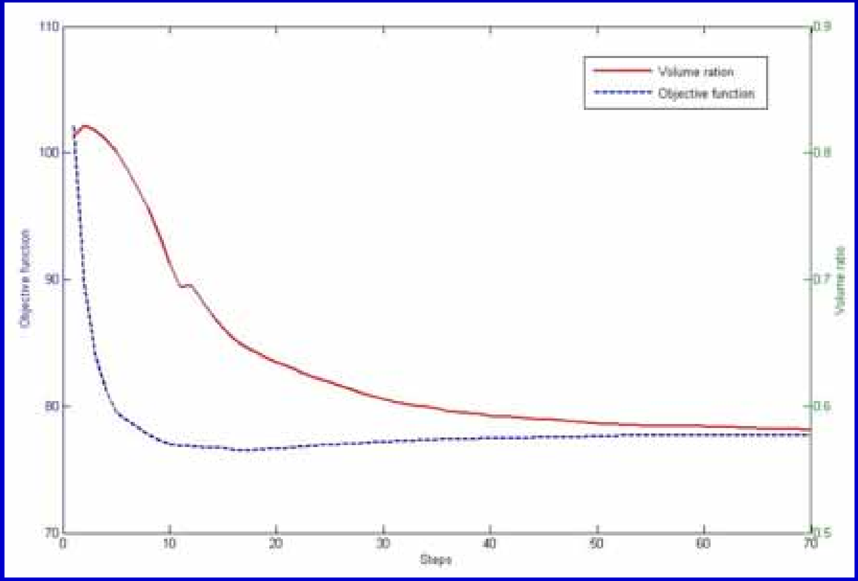


Numerical Examples

Fig.9 Efficiency comparison between the upwind and semi-Lagrange schemes

Type	$J(\Phi)$	N	$T(s)$	$t_{ls}(s)$	$t_{FEM}(s)$
semi-Lagrange	77.69	70	658.06	0.78	7.76
upwind	78.37	6200	3810.98	0.12	8.12

Fig.10 The objective function and the volume ratio for the semi-Lagrange scheme



Numerical Examples

- ***Bridge type structure***

Fig.11 The design problem

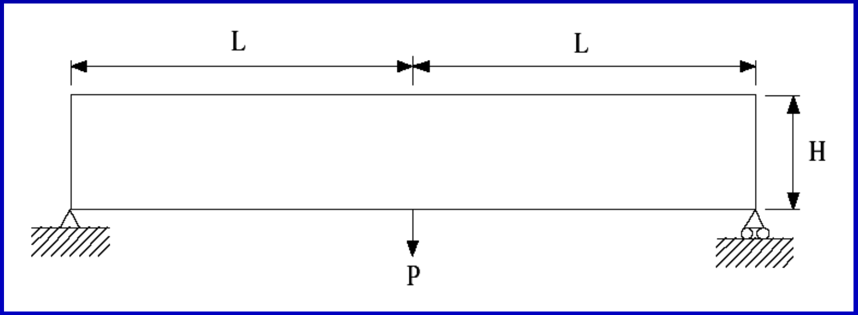
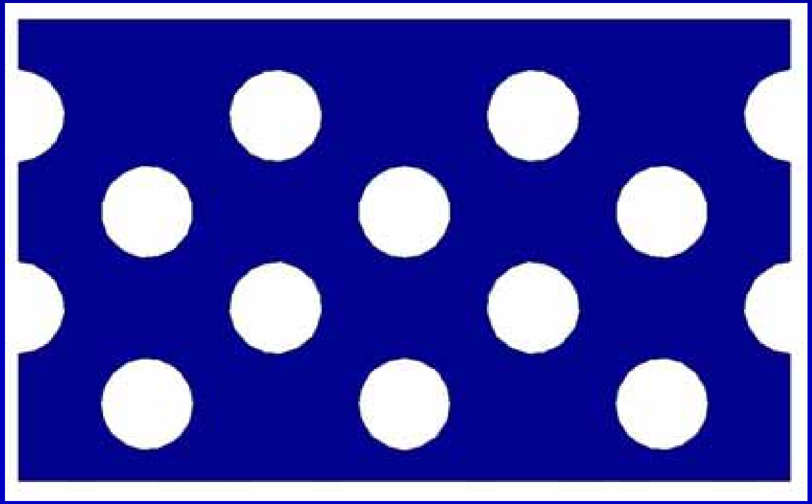
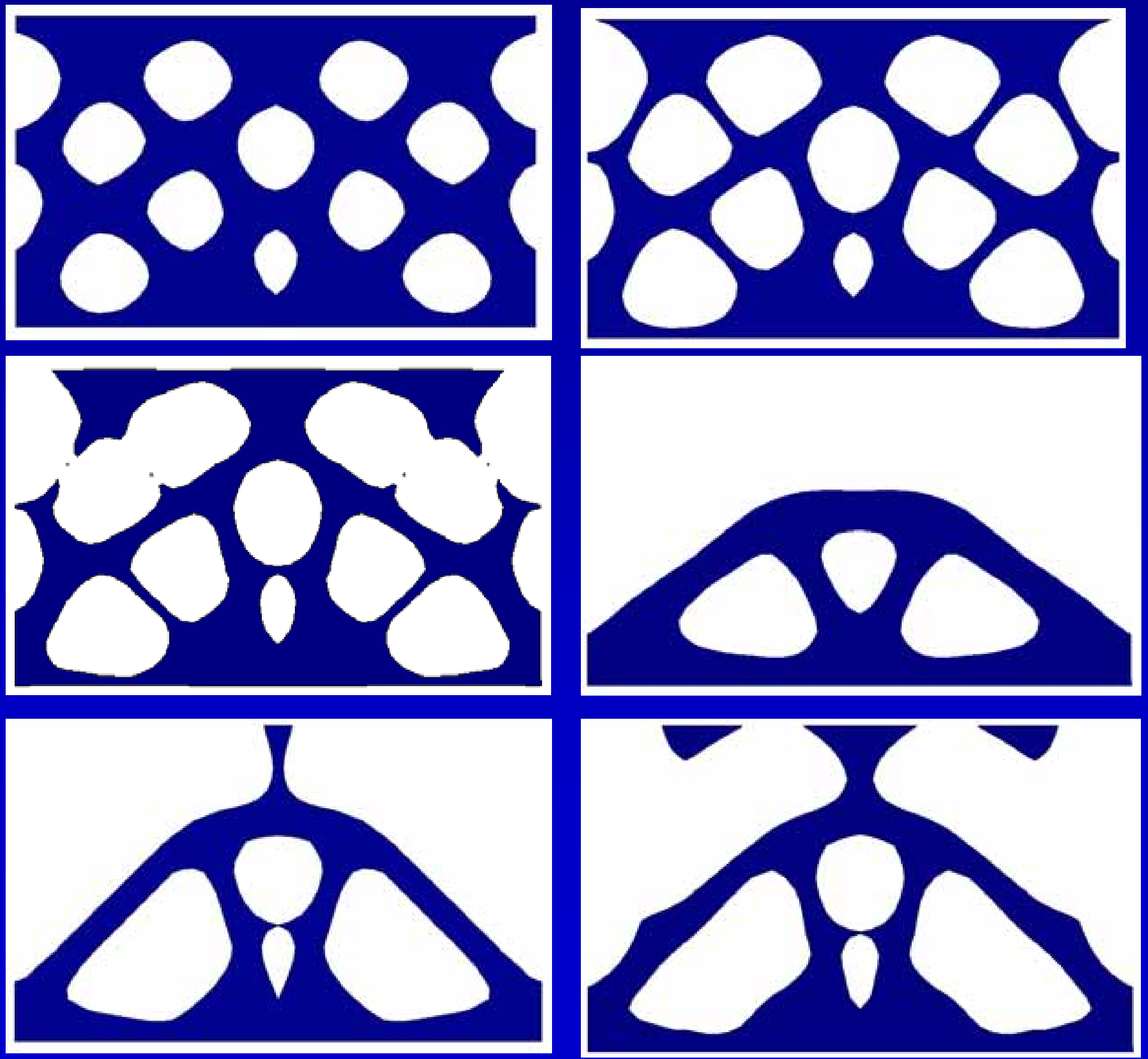


Fig.12 The initial design



Numerical Examples

Fig.13 The optimization process

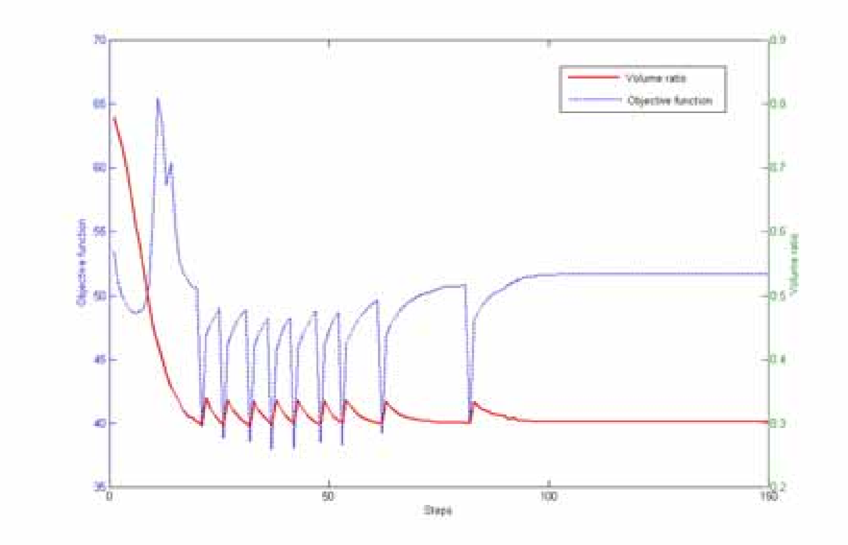


Numerical Examples

Fig.9 Efficiency comparison between the upwind and semi-Lagrange schemes

Type	$J(\Phi)$	N	$T(s)$	$t_{ls}(s)$	$t_{FEM}(s)$
semi-Lagrange	51.54	100	506.78	0.37	2.56
upwind	47.59	1220	1020.98	0.10	2.28

Fig.10 The objective function and the volume ratio for the semi-Lagrange scheme



Conclusion

• *We presented a semi-Lagrange scheme in level set based structural optimization approach to improve the efficiency.*

•*We showed the general principal of semi-Lagrange scheme, strategy to approximate the upstream point, and the ENO interpolation technique.*

• *The results obtained in numerical examples demonstrated that the total time of the optimization process for the semi-Lagrange scheme is much less than that for the explicit upwind scheme.*

The End